

# On a Class of Multiple-Line Directional Couplers\*

C. R. BOYD, JR.†, MEMBER, IRE

**Summary**—Multiple-line directional couplers that utilize only two linearly independent modes of propagation are possible, provided certain restrictions on the maximum coupling are not exceeded. This paper discusses a class of multiple-line directional couplers which may be considered as a generalization of the familiar double-stub four-port directional coupler. The basic design relations for the symmetrical case are developed, and frequency behavior is investigated. Experimental results for an *L*-band six-port hybrid junction are presented and compared with theoretical curves.

Limitations on maximum coupling and fabrication problems will probably confine the number of lines in practical circuits to a small value. Bandwidths of couplers of this class tend to become narrower for the same total coupling off the main line as the number of lines is increased; however, standard techniques for broadbanding are applicable. Experimental results on the six-port hybrid junction agreed well with theory, and the circuit proved to be relatively compact.

## INTRODUCTION

ALTHOUGH four-port directional couplers have been extensively studied in the literature, not much attention has been focused on couplers of more than four ports. This situation undoubtedly results from the fact that combinations of four-port couplers can often be made to serve the function of a single coupler having more than four ports. However, there are some special circumstances, particularly where relatively large coupling or symmetrical behavior is required, for which an array of four-port couplers will not suffice. For such cases, it is desirable to have the alternative of constructing as a unit a directional coupler circuit having more than four ports.

One fairly obvious method for producing such a circuit is to use an array of uniform, electrically parallel transmission lines which are coupled together over some length, and in the proper manner to provide directivity and impedance match. This problem has been analyzed in very general terms by Shelton,<sup>1</sup> with the important special case of the six-port hybrid junction discussed in a later publication by Shelton and Kelleher.<sup>2</sup> However, these works do not describe the results of actual experience with multiple-line couplers, nor is any specific de-

sign information provided to permit construction of a circuit with particular characteristics. This paper is intended to provide the latter information for a particular class of multiple-line directional couplers; namely, those which are based on an array of lines, having uniquely defined phase velocity of propagation, which are periodically interconnected by branch lines. This class may thus be considered as an extension of the familiar "double-stub" four-port directional coupler<sup>3</sup> to a larger number of ports.

In the analysis which follows, an attempt has been made to outline the problem in a manner which compromises simplicity and thoroughness. Thus, the eigenvalue nature of the transmission line array problem is not developed in detail, and the basic design relations are worked out only for the symmetrical case. The frequency dependence of the class of couplers considered is investigated in sufficient detail to permit the behavior of specific cases to be computed with a minimum of effort according to techniques which are fully outlined. Finally, experimental results for a six-port hybrid junction are presented and compared with theoretically expected performance.

## GENERAL CONSIDERATIONS

For the purposes of this analysis, an ideal multiple-line directional coupler will be defined as an  $n$ -port lossless, reciprocal junction having a scattering matrix of the form

$$S = \begin{bmatrix} 0 & \alpha \\ \tilde{\alpha} & 0 \end{bmatrix} \quad (1)$$

where  $\alpha$  is a square matrix of rank  $n/2$ , with  $\tilde{\alpha}$  its transpose. Thus the junction consists of two sets of  $n/2$  ports, where all ports within a given set are mutually isolated. For the type of geometry to be studied, *i.e.*, one involving electrically parallel transmission lines with coupling, it is readily demonstrated on heuristic grounds that each set of ports must comprise one end of the array of lines. The coupling then occurs as a result of the transmission characteristics of the system rather than as the result of controlled reflections; thus "backward" cou-

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† Syracuse University, Syracuse, N. Y.; on leave of absence from the Electronics Laboratory, General Electric Company, Syracuse, N. Y.

<sup>1</sup> J. P. Shelton, Jr., "Multiple-line directional couplers," 1957 IRE NATIONAL CONVENTION RECORD, pt. 1, pp. 254-262.

<sup>2</sup> J. P. Shelton and K. S. Kelleher, "Multiple beams from linear arrays," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-9, pp. 154-161; March, 1961.

<sup>3</sup> C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., p. 309; 1948.

pling similar to that used in the directional coupler described by Firestone,<sup>4</sup> Oliver,<sup>5</sup> and Knechtli<sup>6</sup> is unsuitable.

The eigenvalue nature of the uniform multiconductor transmission line problem has been explored by Pipes<sup>7</sup> and others. In general, it is found that for a lossless system of  $n+1$  parallel conductors, there exist  $n$  distinct velocities of propagation and consequently  $n$  unique, linearly independent, orthogonal (in a power sense) solutions for voltage and current on the lines. For cases in which less than  $n$  distinct velocities of propagation exist, the number of linearly independent solutions remains the same, but modes at degenerate velocities are no longer unique. The homogeneously filled case and the case where one conductor is warped to form a shield which isolates the remaining conductors from each other are ones in which an  $n$ -fold degeneracy can be assumed. The utility of the mode approach, of course, is that a problem involving multiconductor transmission lines can be reduced after rotation to its normal coordinates to a number of problems in simple transmission lines, whose solutions are well known and may sometimes even be written by inspection. It is the eigenvalue problem, therefore, that underlies the "symmetrical analysis" methods that have been used in the study of four-port junctions<sup>8-10</sup> and which will be used here in the description of the multiple-line directional couplers of interest.

Consider, then, an array of  $n$  identical, parallel, isolated transmission lines. Suppose further that a monochromatic traveling wave is impressed on one of these lines, and that the remaining lines are unexcited. A distribution of excitation such as this can be represented as the linear superposition of an unbalanced mode, in which all lines are driven equally against ground, and one or more balanced modes, in which no ground currents flow. For example, the phasors of Fig. 1 depict the situation described when the unbalanced mode and only one balanced mode are used to represent the postulated excitation.

In order to obtain directional coupler behavior of the prescribed sort, it is necessary to couple the lines together over some region; the conditions which must be satisfied by the balanced and unbalanced modes are

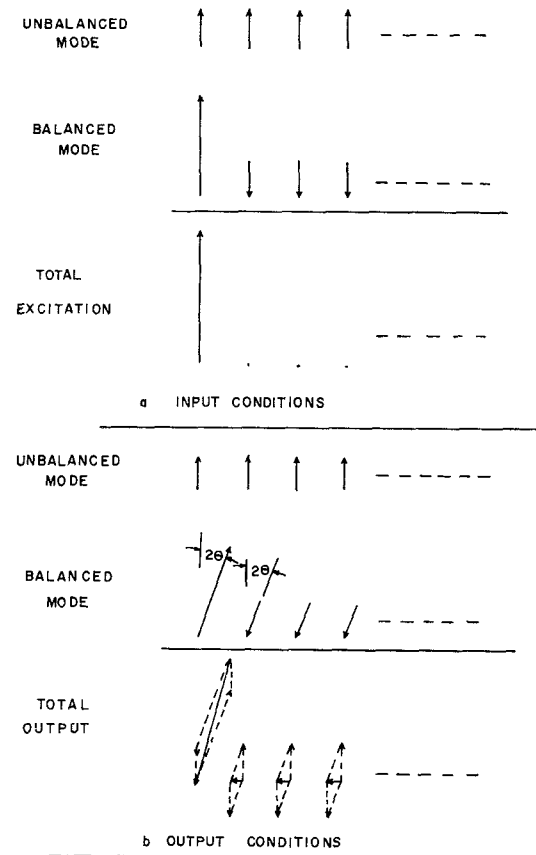


Fig. 1—Mode phasor diagrams.

readily determined. First, the various modes must be independently matched at the input to the coupled region when the output lines are properly terminated, so that no reflected waves are launched, *i.e.*, isolation between input ports is maintained. Also, the electrical geometry must be symmetrical so that symmetry in the coupling is preserved, which implies that conversions between unbalanced and balanced modes do not result from the coupling. Finally, the coupling must produce a differential phase shift between modes propagating through the coupled region so that waves appear on all lines at the output end.

The mode configuration which is basic to the class of directional couplers of interest here is one of the simplest, namely the two-mode system of Fig. 1. If the amplitude of the resultant applied wave is taken as unity, then the amplitudes of the unbalanced mode phasors are each  $1/n$ , and the balanced mode phasors have amplitudes of  $(n-1)/n$  (for the excited line) or  $1/n$  (for the unexcited lines), where  $n$  is the number of transmission lines in the array. Suppose now that these two modes propagate through a coupled region such that no reflections or mode conversions take place, but that a differential phase shift of angle  $2\theta$  occurs. Then the wave amplitude which appears on any line at the output of the coupled region can be found simply by a vector addition of the two mode phasors representing

<sup>4</sup> W. L. Firestone, "Analysis of transmission line directional couplers," *Proc. IRE*, vol. 42, pp. 1529-1538; October, 1954.

<sup>5</sup> B. M. Oliver, "Directional electromagnetic couplers," *Proc. IRE*, vol. 42, pp. 1686-1692; November, 1954.

<sup>6</sup> R. C. Knechtli, "Further analysis of transmission-line directional couplers," *Proc. IRE*, vol. 43, pp. 867-869; July, 1955.

<sup>7</sup> L. A. Pipes, "Matrix theory of multi-conductor transmission lines," *Phil. Mag.*, vol. 24, pp. 97-100; July, 1937.

<sup>8</sup> B. A. Lippman, "The Theory of Directional Couplers," Radiation Lab., Mass. Inst. Tech., Cambridge, Mass., Report No. 860; December 28, 1945.

<sup>9</sup> E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. MTT-4, pp. 75-81; April, 1956.

<sup>10</sup> J. Reed and G. J. Wheeler, "A method of analysis of symmetrical four-port networks," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. MTT-4, pp. 246-252; October, 1956.

the total amplitude on that line. For the two-mode system, only two possibilities exist, *viz.*,

1) Excited line,

$$|e| = \sqrt{1 - \frac{2}{n^2}(n-1)(1 - \cos 2\theta)} \quad (2)$$

2) Unexcited line,

$$|e'| = \sqrt{\frac{2}{n^2}(1 - \cos 2\theta)}. \quad (3)$$

It is a simple matter to verify that power is conserved by this operation, *i.e.*,

$$|e|^2 + (n-1)|e'|^2 = 1. \quad (4)$$

The most conspicuous deficiency of the symmetrical two-mode system is its property that the maximum obtainable coupling falls off steadily as the number of lines increases. The maximum coupling to the auxiliary lines is achieved when  $2\theta = \pi$ , and has the magnitude

$$|e'|_{\max} = \frac{2}{n}. \quad (5)$$

Thus the maximum total amount of power  $P_t$  that can be transferred from the excited line to the auxiliary lines is a monotonically decreasing function of  $n$ ,

$$P_t = Y_0(n-1)|e'|^2 = 4Y_0 \frac{n-1}{n^2} \quad (6)$$

where  $Y_0$  is the characteristic admittance of each line in the group. The limitations imposed by this relationship are illustrated in Fig. 2.

One important special case of the multiple-line directional coupler is that in which the output lines have equal amplitude for excitation on a single input line. The familiar 3-db directional couplers, or 90° hybrid junctions, are two-line systems having this property. Three-line systems so constructed may be defined as six-port hybrid junctions, four-line systems as eight-port hybrid junctions, and so on. The basic condition is that

$$|e'|^2 = \frac{1}{n}. \quad (7)$$

In the two-mode coupler, the value of  $|e'|^2$  has the upper bound given by (5) above. This upper bound is greater than  $1/n$  for  $n < 4$ , equal to  $1/n$  for  $n = 4$ , and less than  $1/n$  for  $n > 4$ . Consequently, two-mode hybrid junctions of more than eight ports are not possible, a fact which has been previously pointed out by Shelton and Kelleher.<sup>2</sup> Hybrid junctions with more than eight ports are possible, of course, by interconnecting an array of four-port or six-port junctions, or by using a larger number of modes in the same general configuration as

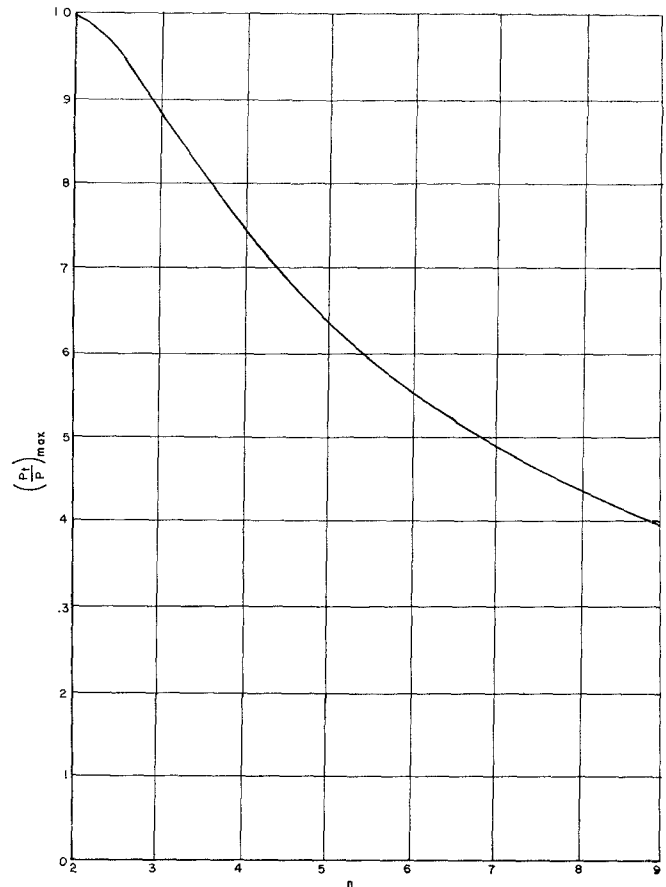


Fig. 2—Dependence of total coupled power on number of lines.

above. For example, a ten-port hybrid junction can be postulated which uses four modes at four different phase shift angles. The physical realization of such a ten-port junction would necessarily be quite different than the two-mode realization to be described at this point.

#### REALIZATION OF THE TWO-MODE COUPLER

Given the basic array of parallel transmission lines, the problem of realization is that of specifying the proper kind of coupling. The existence of four-port couplers is certainly sufficient evidence that the proper kind of coupling can be achieved for two-line systems; reason demands, therefore, that the coupling arrangements of four-port junctions be examined for clues leading to more general structures.

One of the oldest and simplest of four-port junctions is the "double-stub" coupler of Fig. 3. In this junction, the two lines are coupled by a pair of identical quarter-wave branch lines spaced a quarter wave apart. For unbalanced-mode propagation, a virtual open-circuit exists at the mid-point of each of these branches, and the net effect is that of capacitive loading of each of the parallel lines by susceptances equal in magnitude to the characteristic admittance of the branch lines. For balanced-mode propagation a virtual short-circuit exists at the midpoint of each branch, producing an equivalent

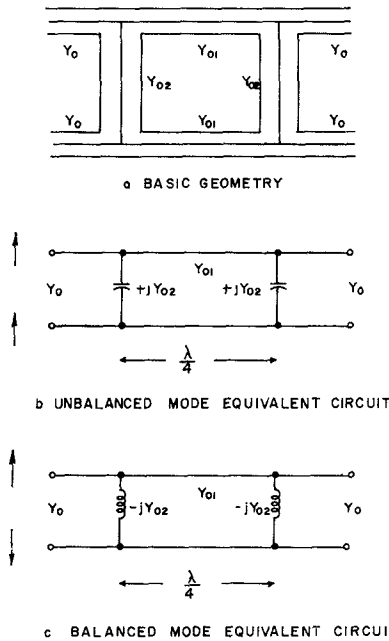


Fig. 3—Four-port double-stub coupler properties.

inductive susceptance of magnitude equal to the branch characteristic impedance. Because of the equal susceptive loading and the quarter-wave spacing of loads, both modes are then simultaneously matched by altering the characteristic admittances of the parallel lines between the branches. Finally, the difference in sign of the loading causes a differential phase shift to occur between the two modes as they propagate through the structure so that directional coupler behavior is obtained.

Suppose now that a third parallel line is added to the system, and that a pair of eighth-wavelength branch lines are connected from it to the midpoints of the existing branch lines. A three-line system is thus produced which preserves the property that a virtual open circuit exists for the unbalanced mode and a virtual short circuit for the balanced mode at the symmetry point of the branch structure. Then the same equivalent circuits can be drawn for the two modes of the three-line system as for the two-line system, and the same impedance-matching conditions apply. A fourth line having eighth-wavelength branches connected to the centers of the branch stars can be added, again without altering the match conditions; this in turn may be followed by a fifth line, etc., up to as many lines as desired. Of course, while the number of lines is in principle unlimited, practical considerations would undoubtedly place restrictions on the complexity of the array.

The band-center design relations for double-stub couplers are well known, and may be immediately extended to the multiple line case. The derivation is so simple, however, that it seems worthwhile to repeat it here so that a particularly useful geometric relationship between design parameters can be presented. Thus, for

the unbalanced mode [see Fig. (3b)], the band-center input match condition is given by

$$Y_0 = jY_{02} + \frac{Y_{01}^2}{Y_0 + jY_{02}} \quad (8)$$

where

$Y_0$  = characteristic admittance of input and output lines

$Y_{01}$  = characteristic admittance of parallel connecting lines

$Y_{02}$  = characteristic admittance of shunt branches.

Solving for  $Y_{01}^2$ ,

$$Y_{01}^2 = Y_0^2 + Y_{02}^2. \quad (9)$$

The same matching condition obviously applies to the balanced mode case. From elementary transmission line theory, the voltage at the output shunt branch for the unbalanced mode is related to the input voltage at band center by

$$V_{out} = -V_{in} \frac{1 + \rho}{1 - \rho} \quad (10)$$

where  $\rho$  is the reflection coefficient at the output branch. However,

$$\frac{1 + \rho}{1 - \rho} = \frac{Y_{01}}{Y_0 + jY_{02}} = \frac{Y_{01}(Y_0 - jY_{02})}{Y_0^2 + Y_{02}^2} \quad (11)$$

and upon substitution according to (9) for the denominator,

$$V_{out} = -jV_{in} \frac{Y_0 - jY_{02}}{Y_{01}}. \quad (12)$$

In polar form, (12) reduces to

$$V_{out} = V_{in} \exp \left\{ -j \left( \pi/2 + \tan^{-1} \frac{Y_{02}}{Y_0} \right) \right\}. \quad (13)$$

The  $\pi/2$  angle is, of course, the result of the quarter-wave spatial separation between input and output branches; however, the additional angle is caused by the shunt loading of the branches, and contributes to the differential phase shift between modes. By corresponding analysis, it is seen that the balanced mode angle has the value  $-\pi/2 + \tan^{-1} Y_{02}/Y_0$ , i.e., the shift caused by shunt loading is in the opposite sense. Since the total differential phase shift angle, defined previously as  $2\theta$ , is the sum of these equal terms, it is concluded that

$$\frac{Y_{02}}{Y_0} = \tan \theta. \quad (14)$$

The relationships of (9) and (14) are all the information that is required for the design of a simple coupler once the angle  $\theta$  has been chosen; and the value of  $\theta$ , of course, is governed by the number of lines and the de-

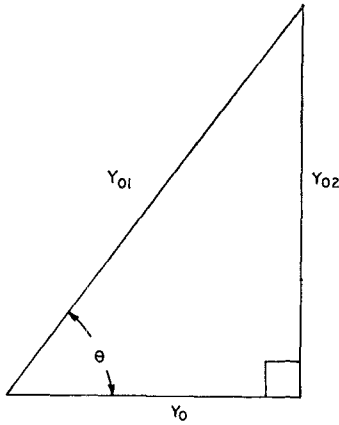


Fig. 4—Triangle relationship for design parameters.

sired degree of coupling, according to (2) and (3). The conclusions of (9) and (14) are easily remembered by observing that the magnitudes  $Y_0$ ,  $Y_{01}$ , and  $Y_{02}$  form a right triangle having  $\theta$  as the included angle of sides  $Y_0$  and  $Y_{01}$ . This triangle is illustrated in Fig. 4.

#### FREQUENCY DEPENDENCE

In order to determine the frequency dependence of the double-stub directional coupler, it is necessary to analyze its circuit properties in somewhat greater depth. A method of analysis which can be used to advantage is to write the short-circuit admittance parameters for each of the two characteristic modes of propagation, then form the  $Y_0$ -normalized admittance matrices for each mode, from which scattering matrices are derived according to the well-known matrix relationship

$$\mathbf{S} = (\mathbf{I} - \mathbf{Y})(\mathbf{I} + \mathbf{Y})^{-1}. \quad (15)$$

The total scattering matrix may then be found by adding appropriate combinations of the unbalanced-mode and balanced-mode scattering matrices. Of course, the number of elements which must be computed is reduced considerably by the symmetry of the junction; from Fig. 1, it can be seen that only two types of output exist for the two-mode system. In addition, two types of reflection must be considered, so that there are only four distinct parameters regardless of the total number of ports. Thus the total scattering matrix is of the form

$$\mathbf{S} = \begin{bmatrix} \gamma & \delta & \delta & \cdots & | & \eta & \xi & \xi & \cdots \\ \delta & \gamma & \delta & \cdots & | & \xi & \eta & \xi & \cdots \\ \delta & \delta & \gamma & \cdots & | & \xi & \xi & \eta & \cdots \\ \vdots & \vdots & \vdots & \ddots & | & \vdots & \vdots & \vdots & \ddots \\ \vdots & \vdots & \vdots & \vdots & | & \vdots & \vdots & \vdots & \vdots \\ \hline \eta & \xi & \xi & \cdots & | & \gamma & \delta & \delta & \cdots \\ \xi & \eta & \xi & \cdots & | & \delta & \gamma & \delta & \cdots \\ \xi & \xi & \eta & \cdots & | & \delta & \delta & \gamma & \cdots \\ \vdots & \vdots & \vdots & \ddots & | & \vdots & \vdots & \vdots & \ddots \\ \vdots & \vdots & \vdots & \vdots & | & \vdots & \vdots & \vdots & \vdots \end{bmatrix}. \quad (16)$$

The short-circuit admittance parameters for the unbalanced and balanced modes are easily determined, and the normalized admittance matrices found to be

$$\mathbf{Y}_{\text{unbal}} = -j \begin{bmatrix} \frac{Y_{01}}{Y_0} \cot \beta l - \frac{Y_{02}}{Y_0} \tan \frac{\beta l}{2} & \frac{Y_{01}}{Y_0 \sin \beta l} \\ \frac{Y_{01}}{Y_0 \sin \beta l} & \frac{Y_{01}}{Y_0} \cot \beta l - \frac{Y_{02}}{Y_0} \tan \frac{\beta l}{2} \end{bmatrix} \quad (17)$$

$$\mathbf{Y}_{\text{bal}} = -j \begin{bmatrix} \frac{Y_{01}}{Y_0} \cot \beta l + \frac{Y_{02}}{Y_0} \cot \frac{\beta l}{2} & \frac{Y_{01}}{Y_0 \sin \beta l} \\ \frac{Y_{01}}{Y_0 \sin \beta l} & \frac{Y_{01}}{Y_0} \cot \beta l + \frac{Y_{02}}{Y_0} \cot \frac{\beta l}{2} \end{bmatrix} \quad (18)$$

where  $\beta$  is the phase constant of the lines and  $l$  the distance between coupling stubs.

From Fig. 4, it is evident that

$$\begin{aligned} \frac{Y_{01}}{Y_0} &= \frac{1}{\cos \theta} \\ \frac{Y_{02}}{Y_0} &= \tan \theta \end{aligned} \quad (19)$$

After some algebraic manipulation, the scattering matrices for the two modes are found to be as follows:

$$\mathbf{S}_{\text{unbal}} = \frac{(a-c)^2 - b^2 - 1 - 2j(a-c)}{[(a-c)^2 - b^2 - 1]^2 + 4(a-c)^2} \cdot \begin{bmatrix} (a-c)^2 + 1 - b^2 & 2jb \\ 2jb & (a-c)^2 + 1 - b^2 \end{bmatrix} \quad (20)$$

$$\mathbf{S}_{\text{bal}} = \frac{(a-d)^2 - b^2 - 1 - 2j(a-d)}{[(a-d)^2 - b^2 - 1]^2 + 4(a-d)^2} \cdot \begin{bmatrix} (a-d)^2 + 1 - b^2 & 2jb \\ 2jb & (a-d)^2 + 1 - b^2 \end{bmatrix} \quad (21)$$

where the quantities  $a$ ,  $b$ ,  $c$ , and  $d$  are defined as

$$\begin{aligned} a &= \frac{\cot \beta l}{\cos \theta} & b &= \frac{1}{\cos \theta \sin \beta l} \\ c &= \tan \theta \tan \frac{\beta l}{2} & d &= -\tan \theta \cot \frac{\beta l}{2}. \end{aligned} \quad (22)$$

A considerable savings in computational labor can be effected by recognizing that when  $\beta l$  differs from  $90^\circ$  by some angle  $\phi$ ,

$$\begin{aligned} (a-c)_{\beta l=90-\phi} &= -(a-d)_{\beta l=90+\phi} \\ b_{\beta l=90-\phi} &= b_{\beta l=90+\phi}. \end{aligned} \quad (23)$$

Then it follows that

$$R_e\{S_{11_{\text{unbal}}}\}_{\beta l=90-\phi} = R_e\{S_{11_{\text{bal}}}\}_{\beta l=90+\phi} \quad (24)$$

$$I_m\{S_{11_{\text{unbal}}}\}_{\beta l=90-\phi} = -I_m\{S_{11_{\text{bal}}}\}_{\beta l=90+\phi} \quad (25)$$

$$R_e\{S_{12_{\text{unbal}}}\}_{\beta l=90-\phi} = -R_e\{S_{12_{\text{bal}}}\}_{\beta l=90+\phi} \quad (26)$$

$$I_m\{S_{12_{\text{unbal}}}\}_{\beta l=90-\phi} = I_m\{S_{12_{\text{bal}}}\}_{\beta l=90+\phi}. \quad (27)$$

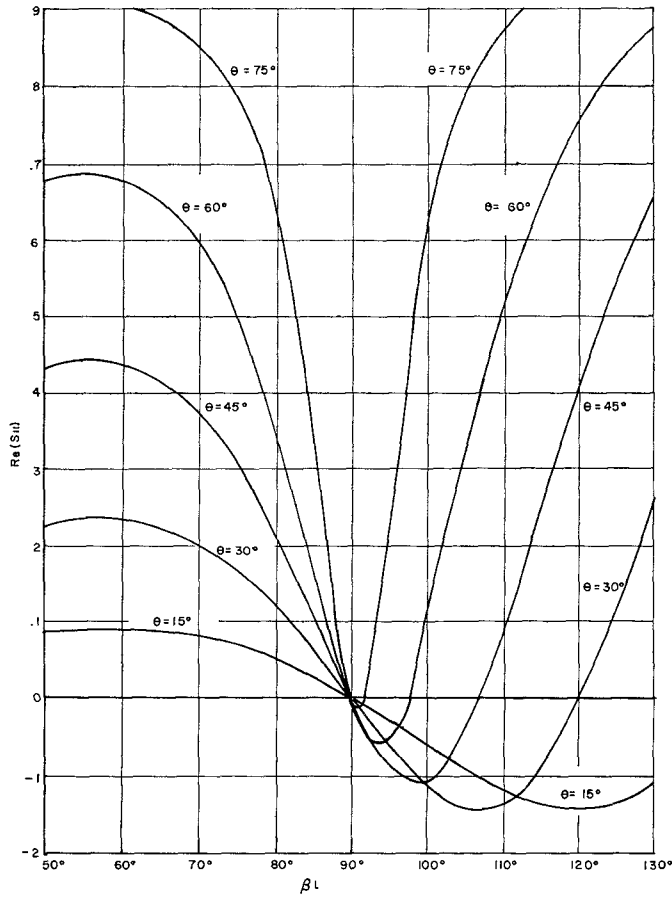


Fig. 5—Real part frequency dependence, unbalanced mode reflection coefficient.

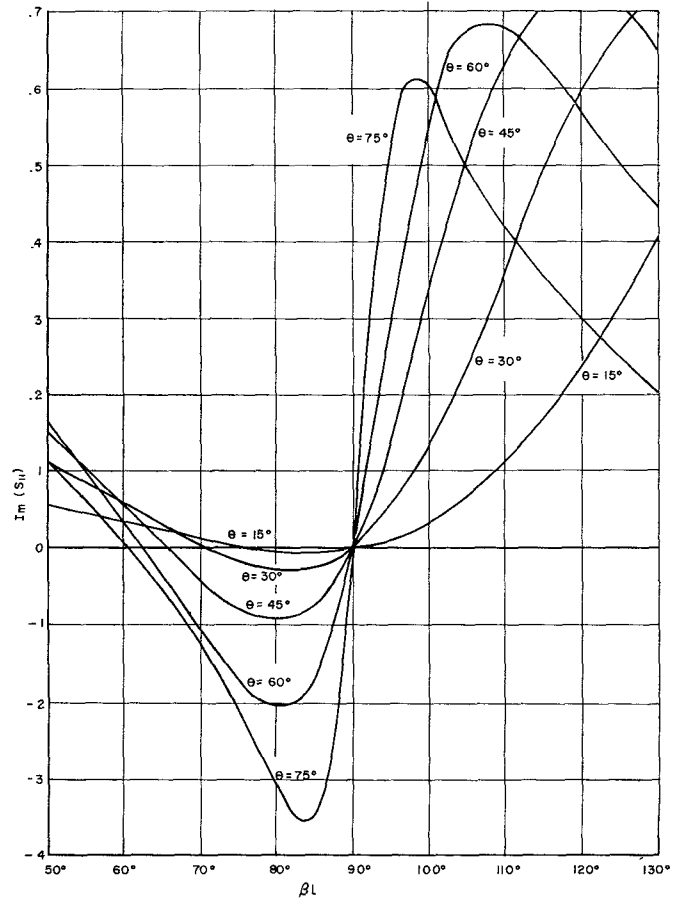


Fig. 6—Imaginary part frequency dependence, unbalanced mode reflection coefficient.

Since the elements of these matrices are of prime importance in determining the over-all circuit behavior, the dependence of the unbalanced mode parameters on the angle  $\beta l$  for selected values of  $\theta$  has been plotted in Figs. 5-8.

Knowing the elements of the mode scattering matrices, it is possible to compute the four independent parameters of the total scattering matrix of (16), which represents the actual performance characteristics of the junction. These are found by vector addition as indicated in Fig. 1, and are given by:

$$\gamma = \frac{1}{n} [S_{11_{unbal}} + (n-1)S_{11_{bal}}] \quad (28)$$

$$\delta = \frac{1}{n} [S_{11_{unbal}} - S_{11_{bal}}] \quad (29)$$

$$\eta = \frac{1}{n} [S_{12_{unbrl}} + (n-1)S_{12_{bal}}] \quad (30)$$

$$\xi = \frac{1}{n} [S_{12_{unbal}} - S_{12_{bal}}]. \quad (31)$$

Thus not only are the transmission coefficients dependent on the number of parallel lines in the system, but the reflection coefficients as well.

#### EXPERIMENTAL RESULTS FOR A SIX-PORT HYBRID JUNCTION

As a means of verifying the theory of stub-type multiple-line directional couplers developed above, a six-port hybrid junction has been constructed and tested. This junction utilizes strip transmission line construction techniques and is consequently quite compact, considering its 1030 Mc center frequency. The basic geometry of the circuit is shown in the sketch of Fig. 9.

The main case of this junction is a brass disk about one-half inch thick and  $3\frac{1}{2}$  inches in diameter. The top and bottom surfaces of this case are machined to provide shallow cavities of  $\frac{1}{8}$ -inch height after fitting on the top and bottom covers. A "sandwich" of two teflon sheets and a brass strip conductor is then placed in each cavity, the conductors having the general shape outlined in the sketch. Thus each cavity contains one of the required star-connected set of branches, while the electrically parallel quarter-wave connecting lines are folded and communicate between the two stars through holes in the internal wall separating the two cavities. The input and output connections are made by means of type-N connectors (not shown in Fig. 9) which are assembled to the top and bottom cover plates.

The design criteria for the six-port hybrid junction are readily determined from the previously developed

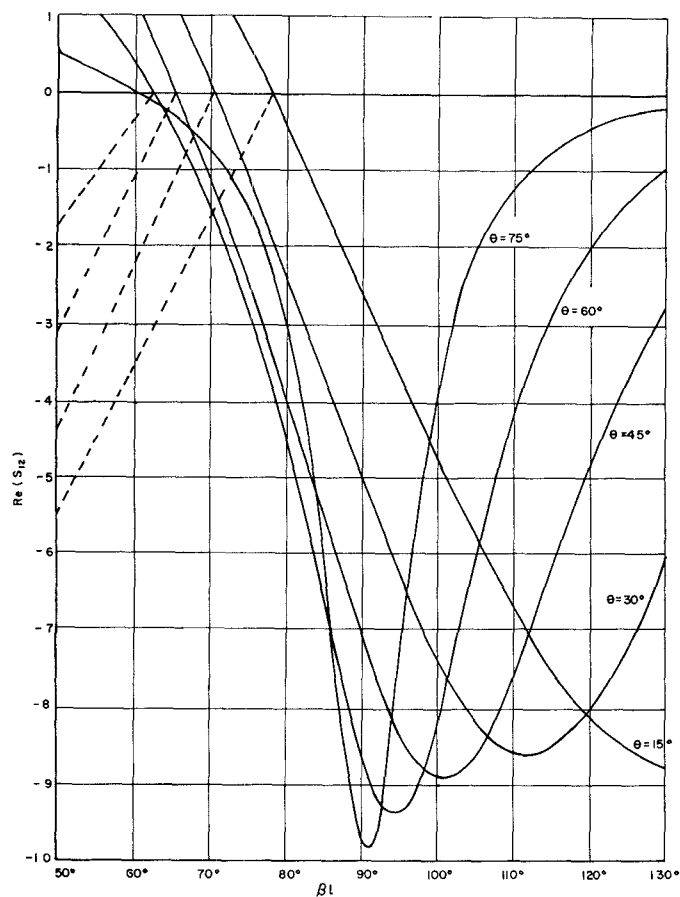


Fig. 7—Real part frequency dependence, unbalanced mode transmission coefficient.

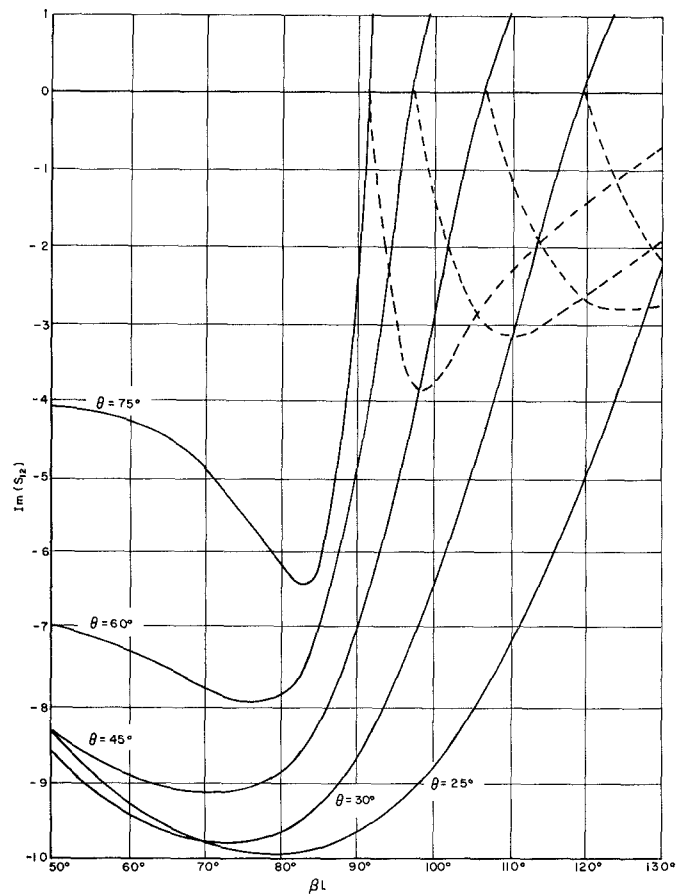


Fig. 8—Imaginary part frequency dependence, unbalanced mode transmission coefficient.

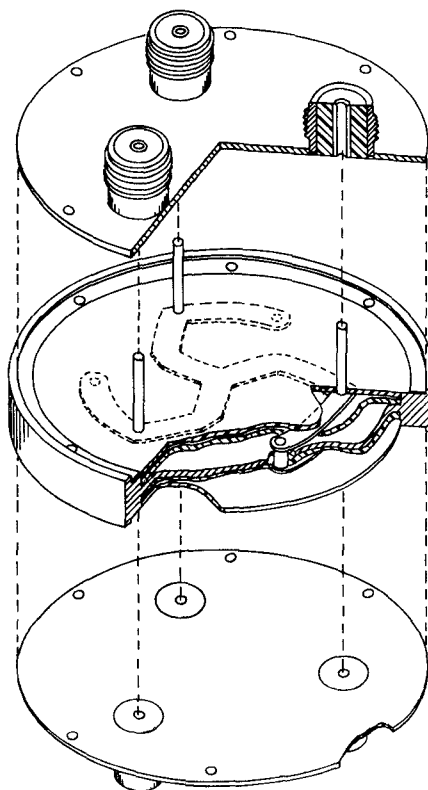


Fig. 9—Exploded view of six-port hybrid junction.

relationships. Using (3) with  $|e'|^2 = \frac{1}{3}$  and  $n=3$ ,

$$\frac{1}{3} = \frac{2}{3}(1 - \cos 2\theta) \quad (15)$$

or

$$\theta = \frac{1}{2} \cos^{-1}(-\frac{1}{2}) = \begin{cases} \pm 60^\circ \\ \pm 120^\circ \end{cases} \quad (16)$$

Taking  $\theta = 60^\circ$ , it follows immediately from Fig. 4, that

$$\begin{aligned} Y_{01} &= 2Y_0 \\ Y_{02} &= \sqrt{3}Y_0. \end{aligned} \quad (17)$$

For  $Y_0 = 20$  millimhos ( $Z_0 = 1/Y_0 = 50$  ohms), the proper values for  $Y_{01}$  and  $Y_{02}$  are 40 millimhos ( $Z_{01} = 25$  ohms) and 34.6 millimhos ( $Z_{02} = 28.9$  ohms), respectively. These were used in constructing the six-port junction described above.

A series of experimental measurements of input VSWR and coupling to all ports was conducted on this junction to determine its behavior over a range of frequencies about band-center. Relative signal amplitudes were determined at low signal level by the IF substitution method, using a calibrated precision attenuator. The data obtained are plotted in Fig. 10, and compared with theoretical performance curves. The latter curves were developed using the computational methods outlined in the previous section of this paper.

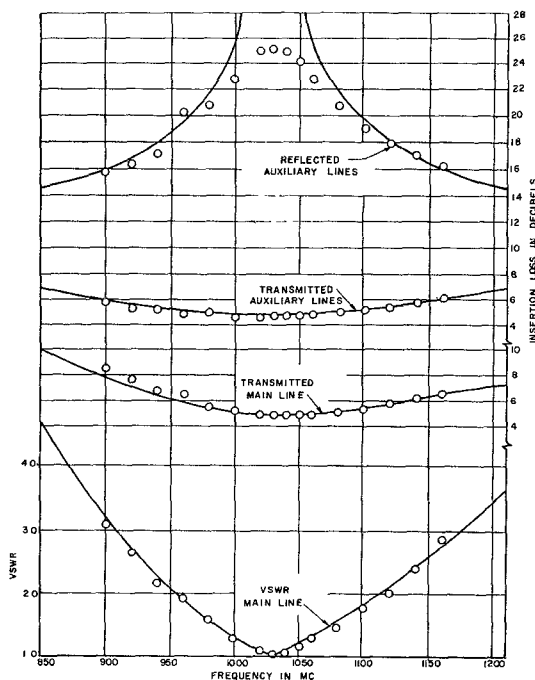


Fig. 10—Theoretical and experimental performance for the six-port hybrid junction.

It is evident that a close correspondence exists between the theoretical and experimental performance, except for the reflections on the auxiliary lines in the vicinity of band-center. The latter deviation is very probably the result of small asymmetries in the junction construction, and could be reduced by sequential modifications.

#### CONCLUSIONS

Within the limitations on the maximum possible coupling, the two-mode system offers a conceptually simple method of obtaining multiple-line directional coupler behavior. Furthermore, the design relations for the double-stub realization are few in number and elementary in form. The physical structure of the double-stub realization also lends itself readily to the fabrication of practical circuits.

Since the double-stub multiple-line directional coupler is a logical extension of the four-port double-stub coupler, it follows that the same techniques for improving bandwidth can be applied. Thus three, four, or more stubs can be used in a binomial or periodic arrangement to achieve the same band-center conditions, but with superior performance off band-center. The binomial arrangement in particular has received a great deal of attention in the literature.

The six-port hybrid junction is perhaps the single most useful multiple-line directional coupler of the class investigated here. In addition to the obvious application of lossless power division by three, it is also possible to use the six-port junction as a circuit for summing power from three coherent sources. By placing a fixed short circuit and two variable-phase short circuits on the three mutually isolated ports of a six-port hybrid junction, a three-port junction can be obtained which will switch power applied at a designated input port to either of the two remaining ports, maintaining a match at the input except under transient conditions. Finally, six-port hybrid junctions can be combined with four-port hybrid junctions to build up larger arrays which may be of use in antenna systems.<sup>2</sup>

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